Mode-independent fuzzy fault-tolerant variable sampling stabilization of nonlinear networked systems with both
time-varying and random delays

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Abstract

This paper develops a fault-tolerant variable sampling control (VSC) scheme for a class of nonlinear networked control systems (NCSs) with time-varying state and random network delays. An uncertain continuous Takagi–Sugeno (T–S) fuzzy system with both state and input varying delays, in the presence of possible actuator faults, is obtained equivalently on the basis of the input delay methodology. A tighter bounding lemma is proposed so as to gain less conservative closed-loop stability criteria. Delay-dependent conditions in terms of linear matrix inequalities are derived for the mode-independent fault-tolerant stabilizing controller of the resulting Markovian network-based system by employing a novel stochastic Lyapunov–Krasovskii (L–K) functional. An illustrative example is simulated to show the validity of the obtained results.

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1. Introduction

A major trend in modern industrial systems is to synthesize communication, control and computing (3C) into different levels of automation processes. It is widely recognized that network-based control is a promising and challenging research field. A feedback system wherein the control loops are closed through a real-time network is called a networked control system (NCS), of which the defining feature is that information is exchanged using a network among system components \cite{36}. When distributed sensor and actuator components communicate with a remote sampling controller component over a multipurpose network, enhanced techniques are needed for stability analysis, controller synthesis and fault tolerance \cite{9,12}. A number of results have been reported for the robust networked controller design, e.g., \cite{4,13,15,25,29,30,35}. Due to imperfect band-limited channels, shared data network connection is not so reliable as traditional point-to-point connection. And it has some conspicuous traits: randomly variable induced delays that depend on highly variable network conditions such as congestion and QoS, possibly non-periodic sampling intervals
An NCS can be transformed into a continuous system based on the input delay approach originally intended for sampled data systems [6], i.e., representing the digital control law as a delayed continuous-time control as follows: 

$$u(t_k) = u(t - (t - t_k)) + u(t - D(t)).$$

An advantage of characterizing NCSs as a delay differential equation over some other modeling processes such as the discrete-time system approach and the hybrid system approach is that the random induced delays can be directly allowed bigger than one sampling interval and this case can be uniformly treated. So we can model NCSs as systems with time-varying input delay generally. More main features of our study are discussed in the following.

In [23], input-delay-based fault-tolerant control of NCSs with periodic sampling against stochastic actuator failure was studied. However, the controlled plant is a linear system without state delays. Networked control of nonlinear systems with constant delay is investigated based on T–S fuzzy model by the authors in [35], but possible actuator failures are not taken into consideration. Therefore, the fuzzy fault-tolerant control problem for nonlinear NCSs with time-varying recycling and random induced delays deserves to be settled reflecting the actual Markovian network situation more precisely.

Another main motivation of the present work is that we need to develop a fault-tolerant VSC scheme for NCSs with interval varying delay. Its goal is to find an aperiodic sampling controller such that the closed-loop nonlinear NCS is asymptotically stable in both normal case and actuator-fault case. The nonuniform sampled-data control scheme only requires that the interval between two consecutive sampling instants should be no longer than a given upper bound. This point relaxes the stringent demand on the periodic sampling of complex sampled data control systems taking a hybrid setting in nature. Although a reliable control study on fuzzy systems in a constant delay under variable sampling without consideration of the network environment was reported in [28], the loss of effectiveness factor they considered takes only 0 or 1. In this paper, the fault factor coefficient can take any value in the interval [0,1]. To the best of our knowledge, the issue of fuzzy fault-tolerant VSC for nonlinear NCSs with a time-varying state delay in the presence of possible actuator faults, which are very challenging and of great importance, has not been investigated in the existing literature.

Moreover, in our current paper, during the process of obtaining delay-dependent stochastic stability criteria for closed-loop network-based systems subject to actuator faults, the introduction of uncorrelated augmented matrix items in the L–K functional reduces the computational burden other than involving the free weighting matrix technique and the model transformation, which is an improvement compared with the existing methodologies. An important reciprocally convex combination lemma is also proposed in Section 3 to avoid bounding some integral terms loosely, with the conservatism of previous results is reduced.

The rest of the paper is organized as follows. The time-varying delay modeling in terms of a feasible continuous-time discrete-state Markov process for variable sampling nonlinear NCSs based on T–S fuzzy logic is explored in Section 2. Followed by preliminary lemmas, a robust reliable state feedback stabilizing controller under variable sampling is designed for the transformed uncertain T–S fuzzy model with mixed varying delays in Section 3. Section 4 provides simulation results to demonstrate the effectiveness of the proposed method. Finally, concluding remarks are presented in Section 5.

Notation: The notation used in this paper is standard. The identity and zero matrices of appropriate dimensions are denoted by $I$ and $0$, respectively. $X^T$ stands for the transpose of matrix $X$. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. $\mathbb{N}$ denotes the set of nonnegative integers. For a square matrix $M$, $He(M)$ is defined as $He(M) = M + M^T$. The notation $*$ always represents the block entry induced by symmetry. $E[\cdot]$ stands for the mathematical expectation.
2. Problem formulation

T–S fuzzy model can be regarded as a universal approximator of a most general nonlinear system. Without loss of generality, affine nonlinear classes representing many control systems in real world are our scope in this work. Consider the following nonlinear system with a time-varying delay in the state

\[ \dot{x}(t) = f(x(t), x(t - d(t))) + g(x(t))u(t), \quad x(t) = \varphi(t), \quad t \in [t_0 - d_2, t_0] \]  

(1)

where \( x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m \) denote the state and control input of the system, respectively. We assume that \( f(x(t), x(t - d(t))) \) and \( g(x(t)) \) are sufficiently smooth in domains \( \mathbb{D} \times \mathbb{D} \subset \mathbb{R}^n \times \mathbb{R}^n \) and \( \mathbb{D} \subset \mathbb{R}^n \) respectively, \( f(0, 0) = 0 \), and \( d(t) \) is an interval-bounded and differential time varying delay in state such that \( 0 \leq d_1 \leq d(t) \leq d_2, \dot{d}(t) \leq \mu, d_1, d_2 \) and \( \mu \) are given or estimated scalars. \( t_0 \) is the initial instant and \( \varphi(t) \) is a continuously real-valued initial function vector.

The nonlinear system (1) can be represented by some simple local linear dynamic systems with their linguistic description as

**Plant rule i:** IF \( \zeta_1(t) \) is \( M_{i1} \) and \( \ldots \) and \( \zeta_n(t) \) is \( M_{in} \), THEN

\[ \dot{x}(t) = A_i x(t) + A_{id} x(t - d(t)) + B_i u(t) \]  

(2)

where \( \zeta_j(t) \) are the premise variables, \( M_{ij} \) is a fuzzy set, \( i = 1, 2, \ldots, r; \quad j = 1, 2, \ldots, n \). \( r \) is the index number of fuzzy rules, and \( A_j, A_{id}, B_i \) are the known system matrices, delayed-state matrices and input matrices, respectively.

By using singleton fuzzifier, product inference, and center-average defuzzifier, the global dynamics of the T–S system (2) is described by weighted sums

\[ \dot{x}(t) = \sum_{i=1}^{r} h_i(\zeta(t))[A_i x(t) + A_{id} x(t - d(t)) + B_i u(t)] \]  

(3)

where \( h_i(\zeta(t)) \) denotes the normalized membership function satisfying \( h_i(\zeta(t)) = \omega_i(\zeta(t))/\sum_{i=1}^{r} \omega_i(\zeta(t)); \omega_i(\zeta(t)) = \prod_{j=1}^{n} M_{ij}(\zeta_j(t)) \) and \( M_{ij}(\zeta_j(t)) \) is the grade of membership of \( \zeta_j(t) \) in \( M_{ij} \). Notice the following facts: \( \omega_i(\zeta(t)) \geq 0 \) and \( \sum_{i=1}^{r} \omega_i(\zeta(t)) > 0, \forall t \geq 0 \). Then, we can see that \( h_i(\zeta(t)) \geq 0, \) and \( \sum_{i=1}^{r} h_i(\zeta(t)) = 1, \forall t \geq 0 \).

From (1) and (3) we obtain

\[ \dot{x}(t) = \sum_{i=1}^{r} h_i(\zeta(t))[A_i x(t) + A_{id} x(t - d(t)) + B_i u(t)] + \Delta f + \Delta g \]  

(4)

where \( \Delta f = f(x(t), x(t - d(t))) - \sum_{i=1}^{r} h_i(\zeta(t))[A_i x(t) + A_{id} x(t - d(t))] \), \( \Delta g = g(x(t))u(t) - \sum_{i=1}^{r} h_i(\zeta(t))B_i u(t) \); \( \Delta f + \Delta g \) is the approximation error between the nonlinear model (1) and the global fuzzy model (3).

In the following, we will denote the time-varying scalar \( h_i(\zeta(t)) = h_i \) to ease the notation. In spite of the fact that the sensor–controller delay and controller–actuator delay can be lumped together for time-invariant controllers [36], no corresponding result is found for time-varying controllers including fuzzy controllers. In this paper, to facilitate the time-varying fuzzy controller design, the one-channel network-based configuration in Fig. 1 is used.

Before describing the controller, we make the following nonrestrictive assumptions: (A1) The controller and the actuator are event-driven. The sensor is time-driven even though the plant outputs are sampled possibly aperiodically. The clocks among them are synchronized. (A2) The data are transmitted with a single packet. (A3) Quantization is ignored to focus our attention on the effects of data sampling, network delay, or packet dropouts on the stability of the resulting closed-loop NCSs. (A4) The system’s states are available at every instant to adopt state feedback. (A5) The extreme case that all the actuators are outage is not considered because it is impossible to control the system effectively when it operates in the open loop.

Since there exist the communication delay \( \tau_{sc} \) between the sensor and the controller and computational delay \( \tau_c \) in the controller, which is shown in Fig. 1, the following memoryless state feedback T–S fuzzy model-based control law
is employed for the system (4) by utilizing the idea of parallel distributed compensation (PDC), in which the same fuzzy sets with the fuzzy model are shared for the designed fuzzy controller in the premises

*Control rule i*: IF $\xi_1(t)$ is $M_{i1}$ and ... and $\xi_n(t)$ is $M_{in}$, THEN

$$u(t^+) = K_i x(t - \tau_k^{sc} - \tau_c^k), \quad t \in [t_k + \tau_k^{sc} + \tau_c^k, k \in \mathbb{N}]$$

(5)

where $t_k$ is the sampling instant, $K_i, i = 1, 2, \ldots, r$ are controller gains to be determined and $u(t^+) = \lim_{t \to t^+} u(t)$.

We assume that $u(t) = 0$ before the first control signal reaches the plant. Analogous to (3), the defuzzified output of the PDC controller is given by

$$u(t) = K(t) x(t - \tau_k^{sc} - \tau_c^k) = \sum_{i=1}^{r} h_i K_i x(t_k), \quad t \in [t_k + \tau_k^{sc} + \tau_c^k, k \in \mathbb{N}]$$

(6)

The actuator fault model is described as in [2]

$$u^F(t) = Lu(t)$$

(7)

where $L \in \mathbb{R} = \{\text{diag}\{l_1, \ldots, l_m\}$, $l_q \in [0, 1]$, $q = 1, 2, \ldots, m$. $L$ denotes the fault extent matrix. $l_q = 0$ means that the $q$th system actuator is invalid, $l_q \in (0, 1)$ implies that the $q$th actuator is partial of loss of effectiveness (LOE) and $l_q = 1$ describes that the $q$th actuator operates normally.

Next, substituting (6) and (7) into (4) admits the closed-loop global fuzzy system similar to [15] with possible actuator failures

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i x(t) + A_{id} x(t - d(t)) + B_i L K_j x(t_k)] + \Delta f + \Delta g$$

(8)

for $t \in [t_k + \tau^k, t_{k+1} + \tau^{k+1}]$, $k \in \mathbb{N}$ where $0 < \tau^k = \tau_3^k + \tau_c^k \leq \tau_{MAID}$ denotes the $k$th entire randomly induced delay from the instant $t_k$ sensors sample sensing data from the plant to the instant actuators send action data to the plant, and $\tau_{MAID}$ is the maximally allowable induced delay which can be determined by practical design; $x(t_k)$ is the state vector of the plant sampled at $t_k$, which is a piecewise constant function obtained using a zero-order holder (ZOH). It is worth pointing out that here the sampling interval may be not a fixed period, but a variable interval with both lower bound and upper bound satisfying the following relation:

$$T(k + 1) = t_{k+1} - t_k, \quad 0 \leq T_1 \leq T(k + 1) \leq T_2, \quad \forall k \in \mathbb{N}$$

(9)

Later, assume the initial time $t_0 = 0$, and then the equality $t_k = \sum_{j=1}^{k} T(j)$ holds.

Suppose that there exist known real constant matrices $D_i, E_i, E_{i1}$ and $E_{i2}(i = 1, 2, \ldots, r)$ of appropriate dimensions such that

$$\Delta f = \sum_{i=1}^{r} h_i D_i F_i(t) [E_i x(t) + E_{i1} x(t - d(t))], \quad \Delta g = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j D_i F_i(t) E_{i2} L K_j x(t_k)$$

(10)
where \( F_i(t) \) are unknown time varying matrix functions with Lebesgue measurable elements satisfying
\[
F_i(t) F_i(t) \leq I, \quad i = 1, 2, \ldots, r; \quad \forall t > 0. \tag{11}
\]

Therefore (8) can be rewritten for \( t \in [t_k + \tau^k, t_{k+1} + \tau^{k+1}] \), \( k \in \mathbb{N} \) as
\[
\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i(t)x(t) + A_{id}(t)x(t - d(t)) + B_i(t)LK_j x(t_k)]
\tag{12}
\]
where \([A_i(t), A_{id}(t), B_i(t)] = [A_i, A_{id}, B_i] + D_i F_i(t)[E_i, E_{i1}, E_{i2}]\).

**Remark 1.** Note that \( x(t_k) = x(t - (t - t_k)) \), define \( \tau(t) = t - t_k, t \in [t_k + \tau^k, t_{k+1} + \tau^{k+1}], k \in \mathbb{N} \), and one can reformulate (12) as
\[
\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i(t)x(t) + A_{id}(t)x(t - d(t)) + B_i(t)LK_j x(t - \tau(t))]
\tag{13}
\]
\[x(t) = \phi(t), \quad t \in [-\max\{\tau_2, d_2\}, 0] \]

Here, we have assumed that the difference of consecutive delays is less than one sampling interval, i.e., \( T(k + 1) > \tau^k - \tau^{k+1}, k \in \mathbb{N} \). And this assumption does not contradict with the case of induced delay larger than one sampling interval.

It is clear that
\[
\tau_1 \leq \tau(t) \leq T(k + 1) + \tau^{k+1} \leq \tau_2 \tag{14}
\]
where \( \tau_1 = \min_{k \in \mathbb{N}} \{\tau^k\} \geq 0, \quad \tau_2 = \max_{k \in \mathbb{N}} \{T(k + 1) + \tau^{k+1}\} \leq T_2 + \tau_{MAID} \) are two estimated constants. So the system (12) is equivalent to the linear system (13) with interval time-varying delays in both state and input. The important superiority of characterizing an NCS as in (13) is that the restriction to induced delays smaller than one sampling interval often made in the discretization approach can be removed. We can also view packet dropouts as a delay quantity. This means that an NCS with a maximum number of consecutive dropouts equal to \( N \) (i.e., the drop number \( d(k) \) at time \( t_k \) satisfies \( d(k) \leq N, \forall k \in \mathbb{N} \)) can still be described by a delay equation like (13), with \( \tau_2 = \max_{k \in \mathbb{N}} \{t_{k+1+N} + \tau^{k+1+N} - t_k\} \leq NT_2 + \tau_{MAID} \) instead [12]. Variable delay in NCS is illustrated in Fig. 2.

Considering the Markovian behavior of the network and following the same line as in [13,14], we can use a Markov process \( \{r(t)\} \) to model the random delay \( \tau^k \). \( \{r(t)\} \) is a continuous-time discrete-state Markov process taking values in a finite set \( \mathcal{T} = \{1, 2, \ldots, M\} \) with \( r(0) \in \mathcal{T} \) and the transition probability matrix given by
\[
P[r(t + \Delta) = j|r(t) = i] = \begin{cases} 
\lambda_{ij} \Delta + o(\Delta), & i \neq j \\
1 + \lambda_{ii} \Delta + o(\Delta), & i = j
\end{cases}
\tag{15}
\]
where $\lambda_{ij} > 0$ is the transition rate from mode $i$ to mode $j$ ($i \neq j$), $\sum_{j=1}^{M} \lambda_{ij} = 0$ and $\lim_{A \to 0} o(\Delta)/\Delta = 0$. Modes of the Markov process are defined as different network load conditions. $\{r(t)\}$ can be regarded without loss of generality as the model of $\tau(t)$ [14], since it can be bounded as follows:

$$
\tau_1 \leq \tau_t(t) \leq \tau(t, r(t)) \leq t_{k+1} - t_k + \tau_{r(t,k+1)} \leq \tau_2
$$

(16)

where $\tau_1 = \min_{\gamma \in \gamma} \{\tau_{\gamma}\}$, $\tau_2 = \max_{\gamma \in \gamma, k \in \mathbb{N}} \{t_{k+1} - t_k + \tau_{\gamma}\}$.

The networked systems described by (13), (16) with a mode-independent T–S fuzzy controller [27] can be recast into

$$
\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i x(t) + A_{id} x(t - d(t)) + B_i L K_j x(t - \tau(t, r(t))) + D_i v(t)]
$$

subject to uncertain feedback

$$
v(t) = F_i(t) g(t)
$$

(18)

$$
g(t) = E_i x(t) + E_{i1} x(t - d(t)) + E_{i2} L K_j x(t - \tau(t, r(t)))
$$

(19)

for $t \in [t_k + \tau_{r(t,k)}$, $t_{k+1} + \tau_{r(t,k+1)}$, $r(t_k) \in \gamma, k \in \mathbb{N}$.

In view of (18) and (19), we have

$$
v^T(t)v(t) \leq [E_i x(t) + E_{i1} x(t - d(t)) + E_{i2} L K_j x(t - \tau(t, r(t)))]^T
$$

$$
\times [E_i x(t) + E_{i1} x(t - d(t)) + E_{i2} L K_j x(t - \tau(t, r(t)))]
$$

(20)

We give some preliminary lemmas before deriving the main results.

**Lemma 1 (Han [10]).** For any constant matrix $U \in \mathbb{R}^{n \times n}, U > 0$, scalar $\alpha > 0$, vector function $\dot{x} : [\alpha, 0] \to \mathbb{R}^n$, such that the following integration is well defined, then

$$
-\alpha \int_{\alpha}^{t} \dot{x}^T(\eta) U \dot{x}(\eta) d\eta \leq \left[ \begin{array}{c} x(t) \\ x(t - \alpha) \end{array} \right]^T \left[ \begin{array}{cc} -U & U \\ 0 & -U \end{array} \right] \left[ \begin{array}{c} x(t) \\ x(t - \alpha) \end{array} \right]
$$

**Lemma 2 (Park et al. [22]).** Let $f_1, f_2, \ldots, f_N : \mathbb{R}^m \to \mathbb{R}$ have nonnegative values in an open subset $D$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of $f_i$ over $D$ satisfies

$$
\min_{\{\beta_i | \beta_i \geq 0, \sum_i \beta_i = 1\}} \sum_i \frac{1}{\beta_i} f_i(t) = \sum_i f_i(t) + \max_{g_i(t)} \sum_{i \neq j} g_{i,j}(t)
$$

subject to

$$
\left\{ \begin{array}{c} g_{i,j} : \mathbb{R}^m \to \mathbb{R}, \\ \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \end{array} \right\}
$$

**Lemma 3 (Guan and Chen [8]).** For any real matrices $X_{ij}$, for $1 \leq i \leq r, 1 \leq j \leq r$, and $S > 0$ with appropriate dimensions, we have

$$
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{l=1}^{r} h_i h_j h_k h_l X_{ij}^T S X_{kl} \leq \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j X_{ij}^T S X_{ij}
$$

Lemma 3 can reduce the computational complexity and computational burden successfully. The next one adapted from [7] is useful to obtain a linear matrix inequality form arising from the original non-convex nonlinear matrix inequalities.
Lemma 4. Let us consider a matrix \( Z < 0 \). Given a symmetrical matrix \( Y \) of appropriate dimensions such that \( Y^T Z Y < 0 \), then \( \exists \lambda \in \mathbb{R} \) such that \( Y^T Z Y \leq -2\lambda Y - \lambda^2 Z^{-1} \).

3. Main results

Before giving the main theorem, we present the following important lemma to be used in the proof to derive the existence of reliable fuzzy VSC for nonlinear NCSs.

Lemma 5. For constant matrices \( T, R = R^T > 0 \), scalars \( d_1 \leq d(t) \leq d_2 \), a vector function \( \dot{x} : [-d_2, -d_1] \rightarrow \mathbb{R}^n \) such that the integration in the following inequality is well defined, then it holds that

\[
(d_1 - d_2) \int_{t-d_2}^{t-d_1} \dot{x}^T(z)R\dot{x}(z)dz \leq \dot{\varphi}^T(t) \begin{bmatrix} 1 & -I & 0 \\ 0 & I & -I \\ 0 & 0 & I \end{bmatrix} W \begin{bmatrix} 1 & -I & 0 \\ 0 & I & -I \\ 0 & 0 & I \end{bmatrix} \dot{\varphi}(t)
\]

where

\[
\dot{\varphi}^T(t) = [x^T(t - d_1) x^T(t - d_2)]
\]

\[
W = \begin{bmatrix} -R & T \\ * & -R \end{bmatrix} \leq 0
\]

Proof. In case of \( d_1 = d_2 \), it is obvious that the equality holds because both sides are equal to zero. In case of \( d_1 \equiv d(t) < d_2 \) or \( d_1 < d(t) \equiv d_2 \), Lemma 5 reduces into Lemma 1 which can be seen by simple manipulation.

We will discuss \( d_1 < d(t) < d_2 \) in the following. By Lemma 1, we can get

\[
(d_1 - d_2) \int_{t-d_2}^{t-d_1} \dot{x}^T(z)R\dot{x}(z)dz = (d_1 - d_2) \int_{t-d(t)}^{t-d_1} \dot{x}^T(z)R\dot{x}(z)dz + (d_1 - d_2) \int_{t-d_2}^{t-d(t)} \dot{x}^T(z)R\dot{x}(z)dz
\]

\[
\leq \frac{d_2 - d_1}{d(t) - d_1} \begin{bmatrix} x(t - d_1) \\ x(t - d(t)) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t - d_1) \\ x(t - d(t)) \end{bmatrix}
\]

\[
+ \frac{d_2 - d_1}{d_2 - d(t)} \begin{bmatrix} x(t - d(t)) \\ x(t - d_2) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t - d(t)) \\ x(t - d_2) \end{bmatrix}
\]

In view of \( d_1 \leq d(t) \leq d_2 \) and \( (d(t) - d_1)/(d_2 - d_1) + (d_2 - d(t))/(d_2 - d_1) = 1 \), applying Lemma 2, we can estimate the reciprocally convex combination as follows:

\[
\frac{d_2 - d_1}{d(t) - d_1} \begin{bmatrix} x(t - d_1) \\ x(t - d(t)) \end{bmatrix}^T \begin{bmatrix} R & -R \\ * & R \end{bmatrix} \begin{bmatrix} x(t - d_1) \\ x(t - d(t)) \end{bmatrix}
\]

\[
+ \frac{d_2 - d_1}{d_2 - d(t)} \begin{bmatrix} x(t - d(t)) \\ x(t - d_2) \end{bmatrix}^T \begin{bmatrix} R & -R \\ * & R \end{bmatrix} \begin{bmatrix} x(t - d(t)) \\ x(t - d_2) \end{bmatrix}
\]

\[
\leq \begin{bmatrix} x(t - d_1) - x(t - d(t)) \\ x(t - d(t)) - x(t - d_2) \end{bmatrix}^T \begin{bmatrix} R & -R \\ * & R \end{bmatrix} \begin{bmatrix} x(t - d_1) - x(t - d(t)) \\ x(t - d(t)) - x(t - d_2) \end{bmatrix}
\]

\[
= \begin{bmatrix} x(t - d_1) \\ x(t - d(t)) \\ x(t - d_2) \end{bmatrix}^T \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} R & -R \\ * & R \end{bmatrix} \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(t - d_1) \\ x(t - d(t)) \\ x(t - d_2) \end{bmatrix}
\]
subject to
\[
\begin{bmatrix}
R & -T \\
* & R
\end{bmatrix} \succeq 0
\]

This completes the proof. \(\square\)

**Theorem 1.** For some given constant \(\mu\), negative scalars \(\lambda_l, l \in \mathbb{N}\), nonnegative scalars \(d_1 \leq d_2, \tau_1 \leq \tau_2\), there exists reliable fuzzy VSC which makes the closed-loop networked control systems stochastically stable, if there exist a scalar \(\hat{\epsilon} > 0\), symmetric positive-definite matrices \(X, \hat{Q}, \hat{R}_1, \hat{S}, \hat{S}_i, i = 1, 2, 3, 4\); and \(\hat{M}_k, k = 1, 2\) of appropriate dimensions such that

\[
\begin{bmatrix}
-\tilde{R}_4 & \tilde{M}_1 \\
* & -\tilde{R}_4
\end{bmatrix} \preceq 0
\]

(21)

\[
\begin{bmatrix}
-\tilde{S}_4 & \tilde{M}_2 \\
* & -\tilde{S}_4
\end{bmatrix} \preceq 0
\]

(22)

\[
\Sigma_{ij} + \Sigma_{ji} < 0, \quad 1 \leq i \leq j \leq r
\]

(23)

\[
\Sigma_{ij} = \begin{bmatrix}
\Psi_{11} & \Psi_{12} \\
* & \Psi_{22}
\end{bmatrix}
\]

(24)

where

\[
\Psi_{11} = \begin{bmatrix}
\tilde{Y} & \tilde{R}_3 & A_i dX & 0 & \tilde{S}_3 & B_i L \tilde{K}_j & 0 & \tilde{z} D_i \\
* & \tilde{z}_22 & -\tilde{M}_1 & 0 & 0 & 0 & 0 \\
* & * & (\mu - 1)\hat{Q} - He(\hat{\Gamma}_1) & \tilde{\Gamma}_1 & 0 & 0 & 0 & 0 \\
* & * & * & \tilde{z}_{44} & 0 & 0 & 0 & 0 \\
* & * & * & * & \tilde{z}_{55} & \tilde{\Gamma}_2 & -\tilde{M}_2 & 0 \\
* & * & * & * & * & * & -He(\hat{\Gamma}_2) & 0 \\
* & * & * & * & * & * & \tilde{z}_{77} & 0 \\
* & * & * & * & * & * & * & -\tilde{\epsilon} I
\end{bmatrix}
\]

\[
\Psi_{12}^T = \begin{bmatrix}
d_1 A_i X & 0 & d_1 A_{id} X & 0 & 0 & d_1 B_i L \tilde{K}_j & 0 & \tilde{z}d_1 D_i \\
d_{12} A_i X & 0 & d_{12} A_{id} X & 0 & 0 & d_{12} B_i L \tilde{K}_j & 0 & \tilde{z}d_{12} D_i \\
\tau_1 A_i X & 0 & \tau_1 A_{id} X & 0 & 0 & \tau_1 B_i L \tilde{K}_j & 0 & \tilde{z}\tau_1 D_i \\
\tau_{12} A_i X & 0 & \tau_{12} A_{id} X & 0 & 0 & \tau_{12} B_i L \tilde{K}_j & 0 & \tilde{z}\tau_{12} D_i \\
E_i X & 0 & E_{i1} X & 0 & 0 & E_{i2} L \tilde{K}_j & 0 & 0
\end{bmatrix}
\]

\[
\Psi_{22} = diag\{\tilde{R}_3 - 2X, \tilde{R}_4 - 2X, \tilde{S}_3 - 2X, \tilde{S}_4 - 2X, -\tilde{\epsilon} I\}
\]

with

\[d_{12} = d_2 - d_1\]

\[\tau_{12} = \tau_2 - \tau_1\]

\[\lambda = \max_{l \in \mathbb{N}} \{-\lambda_l\} > 0\]
Moreover, the mode-independent reliable fuzzy controller gain is $K_j = \tilde{K}_j X^{-1}(j = 1, 2, \ldots, r)$.

**Proof.** First define a new process $\{(x_t, r(t)), t \geq 0\}$ by $x_t(s) = x(t + s), −\tau(t, r(t)) ≤ s ≤ 0$ in order to cast our model into the Markovian framework. Now we are ready to construct a novel stochastic Lyapunov–Krasovskii functional

$$V(x_t, r(t), t) = \sum_{i=1}^{3} V_i(x_t, r(t), t)$$

$$V_1(x_t, r(t)) = V_1(x_t) = x^T(t)P x(t)$$

$$V_2(x_t, r(t), t) = \int_{t-d_1}^{t} x^T(s)Q x(s) ds + \int_{t-d_2}^{t-d_1} x^T(s)R_1 x(s) ds + \int_{t-d_2}^{t-d_1} x^T(s)R_2 x(s) ds$$

$$V_3(x_t, r(t), t) = \int_{t-t_1}^{t} x^T(s)S x(s) ds + \int_{t-t_2}^{t-t_1} x^T(s)S_1 x(s) ds + \int_{t-t_2}^{t-t_1} x^T(s)S_2 x(s) ds$$

where $P, Q, S, R_i, S_i > 0, i = 1, \ldots, 4$, and $x_t = x(t + \theta)$, $\theta \in [-c, 0]$, is an element of the Banach space $C([-c, 0], \mathbb{R}^n)$ of continuous functions from $[-c, 0]$ to $\mathbb{R}^n$ with $c = 2 \max\{t_2, d_2\}$.

Let $A$ be the weak infinitesimal generator of $\{(x_t, r(t)), t \geq 0\}$. Then

$$AV_1(x_t) = 2x^T(t)P x(t)$$

$$AV_2(x_t) = x^T(t)(Q + R_1)x(t) + x^T(t - d_1)(R_2 - R_1)x(t - d_1)$$

$$+ (\mu - 1)x^T(t - d_1)Q x(t - d_1)x(t - d_1) - x^T(t - d_2)R_3 x(t - d_2)$$

$$+ x(t)T (d_1^2 R_3 + d_2^2 R_4) x(t) - d_1 \int_{t-d_1}^{t} \dot{x}(z)R_3 \dot{x}(z) dz - d_2 \int_{t-d_2}^{t} \dot{x}(z)R_4 \dot{x}(z) dz$$

Using Lemma 1, we have

$$-d_1 \int_{t-d_1}^{t} \dot{x}(z)R_3 \dot{x}(z) dz \leq \begin{bmatrix} x(t) \\ x(t - d_1) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 \\ * & -R_3 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - d_1) \end{bmatrix}$$

It follows obviously from Lemma 5 that

$$-d_2 \int_{t-d_2}^{t} \dot{x}(z)R_4 \dot{x}(z) dz \leq \chi_1(t)^T \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} N_1 \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \chi_1(t)$$
where
\[
\chi^T_1(t) = [x^T(t - d_1) x^T(t - d(t)) x^T(t - d_2)]
\]
\[
N_1 = \begin{bmatrix} -R_4 & M_1 \\ * & -R_4 \end{bmatrix} \leq 0
\]

Thus, we get
\[
\begin{aligned}
\mathcal{A}V_2(x_t) &\leq x^T(t)(Q + R_1)x(t) + x^T(t - d(t))(R_2 - R_1)x(t - d(t)) \\
&\quad + (\mu - 1)x^T(t - d(t))Qx(t - d(t)) - x^T(t - d(t))R_2x(t - d(t)) \\
&\quad + \dot{x}(t)^T(d_1^2R_3 + d_2^2R_4)\dot{x}(t) + \left[ \begin{array}{c} x(t) \\ x(t - d_1) \end{array} \right]^T \begin{bmatrix} -R_3 & R_3 \\ * & -R_3 \end{bmatrix} \left[ \begin{array}{c} x(t) \\ x(t - d_1) \end{array} \right]
\end{aligned}
\]
\[
+ \chi^T_1(t) \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} N_1 \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \chi^T_1(t)
\]

Thus, we get
\[
+ \chi^T_1(t)
\]

Noticing that \( \dot{x}(t, r(t)) = 1, t \in (t_k + \tau(t_k), t_{k+1} + \tau(t_{k+1})) \) and \( \lambda_m = -\sum_{m \neq l} \lambda_m \leq 0 \), we have
\[
\begin{aligned}
\mathcal{A} \left( \int_{1-\tau(t-r(t))}^t x^T(s)x(s) ds \right)_{r(t)=l}
&= x^T(t)x(s) + \sum_{m \neq l} \lambda_m \int_{t-\tau(t,m)}^{t} x^T(s)x(s) ds \\
&= x^T(t)x(s) + \sum_{m \neq l} \lambda_m \int_{t-\tau(t,m)}^{t} x^T(s)x(s) ds + \lambda_l \int_{t-\tau(t,l)}^{t} x^T(s)x(s) ds \\
&\leq x^T(t)x(s) + \sum_{m \neq l} \lambda_m \int_{t-\tau(t,m)}^{t} x^T(s)x(s) ds + \lambda_l \int_{t-\tau(t,l)}^{t} x^T(s)x(s) ds \\
&\leq x^T(t)x(s) + \lambda \int_{t-\tau_2}^{t-\tau_1} x^T(s)x(s) ds
\end{aligned}
\]

In the same line as estimating \( \mathcal{A}V_2(x_t) \), it follows easily that, for \( t \in (t_k + \tau^k, t_{k+1} + \tau^{k+1}) \), \( k \in \mathbb{N} \)
\[
\begin{aligned}
\mathcal{A}V_3(x_t, r(t)) &\leq (1 + \lambda \tau_2) x^T(t)S_2x(t) + x^T(t)S_1x(t) + x^T(t - \tau_1)(S_2 - S_1)x(t - \tau_1) \\
&\quad - x^T(t - \tau_2)x(t - \tau_2) + \dot{x}(t)^T(S_2 \tau_1 + \tau_2)S_4)\dot{x}(t) \\
&\quad + \chi^T_2(t) \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} N_2 \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \chi^T_2(t)
\end{aligned}
\]

where
\[
\chi^T_2(t) = [x^T(t - \tau_1) x^T(t - \tau(t, r(t))) x^T(t - \tau_2)]
\]
\[
N_2 = \begin{bmatrix} -S_4 & M_2 \\ * & -S_4 \end{bmatrix} \leq 0
\]
Let
\[ 
\chi^T(t) = [x^T(t) \; \chi_1^T(t) \; \chi_2^T(t) \; v^T(t)]
\]

It follows from Lemma 3 that the inequality holds
\[ 
\mathcal{A}V(t, x_t, r(t)) = \mathcal{A}V_1(t, x_t, r(t)) + \mathcal{A}V_2(t, x_t, r(t)) + \mathcal{A}V_3(t, x_t, r(t))
\]
\[ 
= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j x^T(t)(Q + PA_i + A_i^T P + R_1 - R_3 + (1 + \lambda_{r12})S + S_1 - S_3)x(t) \\
+ 2P(A_{id}x(t - d(t)) + B_l K_j x(t - \tau(t, r(t))) + D_j v(t)) \\
+ x^T(t - d_1)E_{22}x(t - d_1) + x^T(t - d(t))((\mu - 1)Q - 2R_4 - He(M_1))x(t - d(t)) \\
- x^T(t - d_2)(R_2 + R_4)x(t - d_2) + x^T(t - \tau_1)(S_2 - S_1 - S_3 - S_4)x(t - \tau_1) \\
- x^T(t - \tau(t, r(t)))(2S_4 + He(M_2))x(t - \tau(t, r(t))) - x^T(t - \tau_2)(S_2 + S_4)x(t - \tau_2) \\
+ \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i x(t) + A_{id} x(t - d(t)) + B_l K_j x(t - \tau(t, r(t))) + D_j v(t)]^T \Theta \\
\times \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i x(t) + A_{id} x(t - d(t)) + B_l K_j x(t - \tau(t, r(t))) + D_j v(t)] \\
\leq \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \chi_i^T(t) \Omega_{ij} \chi_j(t) \\
= \sum_{i=1}^{r} h_i^2 \chi_i^T(t) \Omega_{ii} \chi_i(t) + \sum_{i=1}^{r-1} \sum_{j=i+1}^{r} h_i h_j \chi_i^T(t) (\Omega_{ij} + \Omega_{ji}) \chi_j(t)
\]

where
\[ 
\Omega_{ij} = \begin{bmatrix}
\Xi_{11} & R_3 & \Xi_{13} & 0 & S_3 & \Xi_{16} & 0 & \Xi_{18} \\
* & \Xi_{22} & R_4 + M_1 & -M_1 & 0 & 0 & 0 & 0 \\
* & * & \Xi_{33} & R_4 + M_1 & 0 & A_{id}^T \Theta B_l K_j & 0 & A_{id}^T \Theta D_i \\
* & * & * & \Xi_{44} & 0 & 0 & 0 & 0 \\
* & * & * & * & \Xi_{55} & S_4 + M_2 & -M_2 & 0 \\
* & * & * & * & * & \Xi_{66} & S_4 + M_2 & (B_l K_j)^T \Theta D_i \\
* & * & * & * & * & * & \Xi_{77} & 0 \\
* & * & * & * & * & * & * & D_i^T \Theta D_i
\end{bmatrix}
\]

with
\[ 
\Xi_{11} = Q + (1 + \lambda_{r12})S + He(PA_i) + R_1 - R_3 + S_1 - S_3 + A_i^T \Theta A_i \\
\Xi_{13} = PA_{id} + A_i^T \Theta A_{id} \\
\Xi_{16} = PB_l K_j + A_i^T \Theta B_l K_j \\
\Xi_{18} = PD_i + A_i^T \Theta D_i \\
\Xi_{22} = R_2 - R_1 - R_3 - R_4 \\
\Xi_{33} = (\mu - 1)Q - 2R_4 - He(M_1) + A_{id}^T \Theta A_{id} \\
\]
\[ \Xi_{44} = -R_2 - R_4 \]
\[ \Xi_{55} = S_2 - S_1 - S_3 - S_4 \]
\[ \Xi_{66} = (B_i L K_j)^T \Theta(B_i L K_j) - 2S_4 - H e(M_2) \]
\[ \Xi_{77} = -S_2 - S_4 \]
\[ \Theta = d_1^2 R_3 + d_{12}^2 R_4 + \tau_1^2 S_3 + \tau_{12}^2 S_4 \]

By \( \mathcal{S} \)-procedure, we can obtain from (20)

\[ A V(t) \leq \sum_{i=1}^{r} h_i \sum_{j=1}^{r} h_j x^T(t) \Phi_{ij} x(t) \leq -\varepsilon \|x(s)\|^2 \]

\[ x(s) = \phi(s), \quad s \in [t - \max\{\tau_2, d_2\}, t], \quad t \in [t_k + \tau^k, t_{k+1} + \tau^{k+1}], \quad k \in \mathbb{N} \]

for \( \varepsilon > 0 \), where

\[ \varepsilon = \min_{l \in \mathbb{N}} \left( \lambda_{\min} \left( -\sum_{i=1}^{r} h_i \sum_{j=1}^{r} h_j \Phi_{ij} \right) \right) > 0 \]

\[ \Phi_{ij} = \Omega_{ij} + \begin{pmatrix}
\varepsilon E_{11}^T E_{11} & \varepsilon E_{11}^T E_{12} L K_j & 0 & 0 \\
* & 0 & 0 & 0 \\
* & * & \varepsilon E_{12}^T E_{12} L K_j & 0 \\
* & * & * & 0 \\
* & * & * & \varepsilon (E_{12} L K_j)^T E_{12} L K_j \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{pmatrix} - \varepsilon I \]

Applying Dynkin’s formula [14] yields

\[ E[V(x(t), r(t), t)] - E[V(x(0), r(0), 0)] = E \left[ \int_0^t \mathcal{A}V(x(s), r(s)) \, ds \right] \leq -\varepsilon E \left[ \int_0^t \|x(s)\|^2 \, ds \right] \]

(36)

In view of \( E[V(x(t), r(t))] \geq 0 \), we can thus obtain

\[ E \left[ \int_0^t \|x(s)\|^2 \, ds \right] \leq \frac{1}{\varepsilon} E[V(x(0), r(0))] \]

(37)

which implies that the closed-loop is stochastically stable if \( \Phi_{ij} < 0 \).

According to Schur complement lemma, \( \Phi_{ij} < 0 \) is equivalent to

\[ \begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
* & \Pi_{22}
\end{bmatrix} < 0 \]

(38)
where

\[
\Pi_{11} = \begin{bmatrix}
Y & R_3 & P_{A_1} & 0 & S_3 & P_{B_i}L_i K_j & 0 & PD_i \\
* & \Xi_{22} & \Gamma_1 & -M_1 & 0 & 0 & 0 & 0 \\
* * & (\mu - 1)Q - He(\Gamma_1) & \Gamma_1 & 0 & 0 & 0 & 0 & 0 \\
* * * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 \\
* * * * & * & * & * & * & * & * & \Xi_{55} \\
* * * * * * & * & * & * & * & * & * & \Xi_{77} \\
* * * * * * * * & * & * & * & * & * & * & -\varepsilon I
\end{bmatrix}
\]

\[
\Pi_{12}^T = \begin{bmatrix}
d_1 A_i & d_1 A_{id} & 0 & 0 & d_1 B_i L_i K_j & 0 & d_1 D_i \\
d_{12} A_i & d_{12} A_{id} & 0 & 0 & d_{12} B_i L_i K_j & 0 & d_{12} D_i \\
\tau_1 A_i & \tau_1 A_{id} & 0 & 0 & \tau_1 B_i L_i K_j & 0 & \tau_1 D_i \\
\tau_{12} A_i & \tau_{12} A_{id} & 0 & 0 & \tau_{12} B_i L_i K_j & 0 & \tau_{12} D_i \\
E_i & E_{i1} & 0 & 0 & E_{i2} L_i K_j & 0 & 0
\end{bmatrix}
\]

\[
\Pi_{22} = \text{diag}\{-R_3^{-1}, -R_4^{-1}, -S_3^{-1}, -S_4^{-1}, -\varepsilon^{-1}I\}
\]

with

\[
Y = Q + He(P A_i) + R_1 - R_3 + S_1 - S_3
\]

\[
\Gamma_1 = R_4 + M_1
\]

\[
\Gamma_2 = S_4 + M_2
\]

Pre- and post-multiply both sides of (38) with

\[
diag\{X, X, X, X, X, X, X, \tilde{\varepsilon}I, I, I, I, I, I\}
\]

where \(X = P^{-1}, \tilde{\varepsilon} = \varepsilon^{-1}\). Introducing \(\tilde{Q} = XQX; \tilde{R}_i = XR_i X, \tilde{S}_i = XS_i X, i = 1, 2, 3, 4\); \(\tilde{K}_j = K_j X, j = 1, 2, \ldots, r; \tilde{M}_k = X M_k X, k = 1, 2, \ldots, \), and applying Lemma 4 yield (23)–(24). Similarly, pre- and post-multiply both sides of \(N_k, k = 1, 2\) with \(diag\{X, X\}\) and its transpose, respectively. Hence the proof is completed. \(\square\)

**Remark 2.** If \(\tilde{S}\) related term is eliminated from \(\tilde{Y}\), the result is consistent with that in the deterministic case. When the derivative information of the varying state delay with respect to \(t\) is unknown, by eliminating \(\tilde{Q}\) we have the corresponding result from Theorem 1. Furthermore, it can degrade into a usual networked control problem for plants without delays if \(\tilde{R}_i\)'s are removed. Even if in this case, it can be expected that the results here are less conservative as the tighter bounding lemma is employed to estimate a derivative term of the functional during the proof.

**Remark 3.** In the previous papers [15,30,35], they used some over bounding technique when estimating some terms \(-\tau_{12}\int_{t_{i-2}}^{t_i} \dot{x}(\alpha) S_4 \dot{x}(\alpha) d\alpha \leq -\tau_{12}\int_{t_0}^{t_i} \dot{x}(\alpha) S_4 \dot{x}(\alpha) d\alpha\), however the useful information \(-\tau_{12}\int_{t_{r-2}}^{t_r} \dot{x}(\alpha) S_4 \dot{x}(\alpha) d\alpha\) is also taken into account in this paper. What is more, the term \(-\tau_{12}\int_{t_{r-2}}^{t_i} \dot{x}(\alpha) S_4 \dot{x}(\alpha) d\alpha\) is not over bounded with \(-\tau_{2} - t + t_0\int_{t_i}^{t_{r-2}} \dot{x}(\alpha) S_4 \dot{x}(\alpha) d\alpha\) as done in [22], but derived directly from the matrix version Lemma 1 of the well known Jensen’s inequality and then with, instead, the tighter bounding Lemma 5. Therefore, the conservatism of the sufficient stability criteria is considerably reduced.

**Remark 4.** If \(d(t) \equiv d\) a constant state delay, and induced delays \(\tau^{(t_k)}\) are also ignored, the variable sampling problem studied is similar to one in [28], however, that paper just considered the outage case of actuator failures. The cases of either partial LOE or total failure are all covered in our present paper.
Remark 5. We have embodied the Markovian switching behavior just on the limit of integral in the constructed LK functional for simplicity. Substituting the common matrices $P$ for $P(r(t))$ makes necessary to be known only the minimal diagonal element of the transition rate matrix, i.e., the maximal modular one of the diagonal entries, with that the offdiagonal elements can be unknown as a meaningful byproduct of the proof.

Remark 6. In some recent references [28,30,35], free weighting matrices or model transformations are introduced in order to bring the flexibility to solving the resultant LMIs. However, too many free weighting matrices employed in the existing methods complicate the system analysis and increase the computational demand. Let us exemplify this aspect below.

In [35], the authors introduced free weighting matrices into the Lyapunov–Krasovskii functional combined with a descriptor model transformation. The augmented vector item was chosen as

$$\dot{\xi}^T(t) = [x^T(t) \; \dot{x}^T(t) \; x^T(t - d) \; x^T(t_k)]$$

(39)

To handle this case, there the free weighting matrices $Y_{1i}(i = 1, 2, \ldots, 6)$ are necessary in order to establish the relationship among the correlated items below

$$\dot{\xi}^T(t) = \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{13} & Y_{16} \\ 0 & 0 \\ Y_{12} & Y_{15} \end{bmatrix} \begin{bmatrix} x(t) - x(t_k) - \int_{t_k}^{t} \dot{x}(s) \, ds \\ - \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i x(t) + A_{id} x(t - d) + B_i K_j x(t_k)] + \dot{x}(t) \end{bmatrix} = 0$$

(40)

However, it is easy to see that $\dot{x}(t)$ can be expressed by $x^T(t), \; x^T(t - d), \; \text{and} \; x^T(t_k)$. Taking into account the inner relationship among the terms, we can replace (39) by $\dot{\xi}^T(t) = [x^T(t) \; x^T(t - d) \; x^T(t_k)]$. In this case, it is naturally unnecessary to introduce free weighting matrices or employ the descriptor model transformation for derivation of the system stability, and the system information is also naturally utilized in the LMIs.

It is seen that neither free weighting matrices nor any model transformations have been introduced in our proof of Theorem 1. This is achieved because only uncorrelated augmented vector terms are used in the construction of the augmented vector $\dot{\xi}^T(t)$ in the process of the proof above.

4. Numerical experiment

In this section, an example is taken to show the validity and the effectiveness of the results proposed in this paper. Consider the nonlinear delay system adapted from [3] described as follows:

$$\dot{x}_1(t) = -x_1(3 - \cos^2 x_2) + x_2 + 0.1 x_1(t - d(t)) + 0.2 x_2(t - d(t)) + u_1$$

$$\dot{x}_2(t) = x_1 - x_2 \sin^2 x_2 + 0.2 x_1(t - d(t)) \sin^2 x_2 - 0.5 x_2(t - d(t)) + 0.5 u_2$$

The membership functions can be obtained from the sector nonlinearity methodology in the following:

$$M_1(x_2(t)) = \sin^2(x_2(t)), \quad M_2(x_2(t)) = \cos^2(x_2(t))$$

Then the nonlinear system can be represented by an uncertain T–S fuzzy model:

\textbf{Rule 1: IF} \; x_2(t) \; \text{is} \; M_1, \; \text{THEN}

$$\dot{x}(t) = (A_1 + \Delta A_1)x(t) + (A_{11} + \Delta A_{11})x(t - d(t)) + (B_1 + \Delta B_1)u(t)$$

\textbf{Rule 2: IF} \; x_2(t) \; \text{is} \; M_2, \; \text{THEN}

$$\dot{x}(t) = (A_2 + \Delta A_2)x(t) + (A_{21} + \Delta A_{21})x(t - d(t)) + (B_2 + \Delta B_2)u(t)$$

where $x(t) = [x_1(t), x_2(t)]^T, \; d(t) = 0.65 + 0.36 \sin(2t), \; F(t) = \sin t$. 
Thus, we have

\[
A_1 = \begin{bmatrix} -2.5 & 1 \\ 1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.5 & 1 \\ 1 & 0 \end{bmatrix}
\]

\[
A_{11} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & -0.5 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.5 \end{bmatrix}
\]

\[
B_1 = B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}
\]

\[
E_1 = E_2 = [1 \ 0], \quad DA_{i1} = DB_i = 0 \ (i = 1, 2)
\]
Assume random delays, take two modes with the transition matrix given by

\[ A = (\lambda_{ij}) = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \]

Given the lower bound 0.1 s, we can know the maximal allowable input delay upper bound is about 0.8 s from Theorem 2.

First, assume that \( L = I \), that is, there is no any actuator fault occurring. By applying Theorem 2, the feasible sampled-data state feedback gains are given

\[ K_1 = \begin{bmatrix} 0.1316 & -0.1318 \\ -0.2713 & 0.0137 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.0883 & -0.1837 \\ -0.4839 & -0.7350 \end{bmatrix} \]
Next, we consider three admissible sets of actuator faults, more specifically, partial failure of both two actuators, total failure of first actuator, and total failure of second actuator, listed as

\[ L = \text{diag}(0.89, 0.12), \quad L = \text{diag}(0, 1), \quad L = \text{diag}(1, 0) \]

In the first case of \( L = \text{diag}(0.89, 0.12) \), if we still adopt the controller gains for the case of no any actuator fault, the state curves will be degraded. Assume the initial conditions \( \varphi(t) = [1.5 \ - 2.5]^T \) in this section, the simulation results for the cases of \( L = I \) and \( L = \text{diag}(0.89, 0.12) \) by using the above normal state feedback fuzzy controller \( u(t) = \sum_{i=1}^{2} M_i K_i x(t_k) \) are shown in Figs. 3 and 4, respectively.

The feasible fault-tolerant state feedback gains are computed respectively for the above three cases as follows:

\[
K_1 = \begin{bmatrix}
0.1479 & -0.1480 \\
-2.2606 & 0.1144
\end{bmatrix}, \quad K_2 = \begin{bmatrix}
0.0992 & -0.2064 \\
-4.0324 & -6.1247
\end{bmatrix}
\]

\[
K_1 = \begin{bmatrix}
0 & 0 \\
-0.3611 & 0.1084
\end{bmatrix}, \quad K_2 = \begin{bmatrix}
0 & 0 \\
-0.4184 & -0.8755
\end{bmatrix}
\]

\[
K_1 = \begin{bmatrix}
0.0144 & -0.2908 \\
0 & 0
\end{bmatrix}, \quad K_2 = \begin{bmatrix}
-0.2161 & -0.6802 \\
0 & 0
\end{bmatrix}
\]

The simulation results of state trajectories in three cases are shown in Figs. 5, 6 and 7, respectively. It is observed from Fig. 5 that the degraded state response in Fig. 4 is recovered to a satisfactory extent, the errors from the origin diminished to zero at \( t=3 \) s around. Figs. 6 and 7 also show that the closed-loop system in two failure cases using the fuzzy controller with integrity still operates well and maintains an acceptable level of performance.

**Remark 7.** Extensions of the current derivation for the T-S fuzzy controller based on more powerful relaxation techniques presented in, e.g., [20,34] are straightforward. The more computational burden is required while the conservativeness is further reduced to a less degree. In addition, the feasible controller solution space in terms of LMIs is enlarged, but this does not always imply the better performance than that resulting from the basic quadratic stabilization condition because only the stabilization goal is pursued regardless of involved optimization in this paper.
5. Conclusion

We have investigated the fault-tolerant fuzzy VSC problem for nonlinear NCSs with variable state delay and Markovian network delay in a continuous time system framework. Closed-loop stability criteria of the networked system in the presence of possible actuator faults have been obtained by a novel Lyapunov functional and a newly proposed bounding lemma. Sufficient mode-independent delay-dependent stabilization conditions have been derived in the LMI form. The validity of the obtained results has been verified by a simulation example. The results in this paper can be extended to the cases in which there exist sensor failures just through moving the fault extent matrix to a different appropriate position in the system model. Besides, the case of Markovian switching matrices in the constructed L–K functional may be considered to further reduce the conservativeness of the common switching matrix case in future. The Markovian jump mode-dependent fuzzy control will make the problem in this paper increasingly sophisticated.

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