LMI approach for global robust stability of Cohen–Grossberg neural networks with multiple delays

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Abstract

In this paper, we investigate the global robust stability of the equilibrium point of a class of Cohen–Grossberg neural networks with multiple delays and uncertainties. The new criteria for the global robust stability are given by way of constructing a suitable Lyapunov functional. The criteria take the form of linear matrix inequality (LMI), and are independent of the amplification function. Compared with the other robust stability results, they turn out to be less restrictive. In addition, all results are established without assuming any symmetry of the interconnecting matrix, and the differentiability and monotonicity of activation functions. A simulation example is also given to illustrate the effectiveness of our results.

Keywords: Cohen–Grossberg neural networks; Global robust stability; Multiple delays; Linear matrix inequality

1. Introduction

Since Cohen and Grossberg proposed a class of neural networks in 1983 [4], this model has received increasing interest [14,19,10] due to its promising potential for applications in classification, parallel computing, associative memory, especially in solving some optimization problems [16]. For the Cohen–Grossberg neural networks, many sufficient conditions for the global asymptotic stability or global exponential stability have been obtained. By Lyapunov functional, [3,17] established some criteria for the globally asymptotic stability of this model. In [12], the absolute and exponential stability was studied and an estimate of the rate of convergence was provided. In [18], the global stability of Cohen–Grossberg neural networks was derived under the assumption that the interconnecting matrix was symmetric, when time delays satisfied a computable bound condition. However, it is very difficult to realize absolute symmetry of the interconnecting structure, since the symmetry may often be altered by unavoidable uncertainties due to modeling errors, external disturbances and parameter perturbations during the implementation on very large scale integration (VLSI) chips. On the other hand, due to the finite speeds of switching and transmission of signals, time delays inevitably exist in a working network. Thus, it is very important to consider the influences of time delays and uncertainties when we analyze the stability of neural networks. In recent years, considerable efforts have been devoted to the robust stability analysis of Hopfield neural networks [5,6,11]. However, there are few existing results on the global robust stability for the Cohen–Grossberg neural networks. In [2], Chen and Rong investigated the following delayed Cohen–Grossberg neural networks

\[ \dot{u}_i(t) = -a_i u_i(t) \left( b_i(u_i) - \sum_{j=1}^{n} a_{ij} g_j(u_j(t)) - \sum_{j=1}^{n} h_{ij} f_j(u_j(t - \tau_{ij}(t))) + d_i \right), \quad i = 1, 2, \ldots, n \]

by defining \( \bar{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij} \), \( \bar{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij} \), and \( \bar{h}_{ij} \leq h_{ij} \leq \overline{h}_{ij} \). It should be noted that the criteria in [2] were obtained by giving the absolute values of the upper/below bounds of the interconnecting terms.

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Although these criteria were explicit and easy to verify generally, they often neglected the signs of the terms in the interconnecting matrices, and thus, the difference between the neuronal excitatory and neuronal inhibitory might be ignored. In [7,9], Ji and Zhang analyzed the global robust stability of Cohen–Grossberg neural networks with multiple delays and perturbations of interconnecting weights, and obtained the sufficient conditions for the global robust stability of this model. However, the results given by [7,9] comprised the uncertainties \( \Delta T_k \in \mathbb{R}^{n \times n}, k = 0, \ldots, K \), or amplification function \( a_i(\cdot) \), thus the practicability of the theorem and corollary was reduced.

Aiming at this case, based on the assumption that the uncertainties are norm-bounded, we will give the robust stability criteria expressed by linear matrix inequality (LMI) in this paper. The LMI approach has the advantage that it can be solved numerically and very effectively using the interior-point method. Furthermore, the criteria are independent of the amplification function. In contrast to the results dependent on the amplification function in [8], the criteria in this paper turn out to be less restrictive. The paper is organized as follows. In Section 2, the network model will be described and two assumptions will be given as the basis of later sections. By constructing a suitable Lyapunov functional, we will establish the sufficient conditions for the robust stability of this model in Section 3. Simulation examples will be given in Section 4 to demonstrate the effectiveness of the new results. Finally, Section 5 will summarize the results and future work.

2. Model description and preliminaries

We consider a class of generalized neural networks with time-varying delays described by the following nonlinear differential equations:

\[
\dot{u}(t) = -\varphi(u(t)) \left[ \beta(u(t)) - T_0 G(u(t)) - \sum_{k=1}^{K} T_k G(u(t - \tau_k)) + D \right],
\]

(1)

where \( u = (u_1, \ldots, u_n)^T \in \mathbb{R}^n \), \( \varphi(u) = \text{diag}(\varphi_1(u_1), \ldots, \varphi_n(u_n)) \), \( \beta(u) = [\beta_1(u_1), \ldots, \beta_n(u_n)]^T \), \( G(u) = [g_1(u_1), \ldots, g_n(u_n)]^T \), and \( D = [d_1, \ldots, d_n]^T \). Amplification function \( \varphi_i(\cdot) \) is positive, continuous and bounded, behaved function \( \beta_i(\cdot) \) and activation function \( g_i(\cdot) \) are subject to certain conditions to be specified later. \( d_i \) denotes the \( i \)th external input. \( T_0 = [t_{0ij}] \in \mathbb{R}^{m \times n} \) denotes that part of the interconnecting structure which is not associated with delay, \( T_k = [t_{kij}] \in \mathbb{R}^{m \times n} \) denotes that part of the interconnecting structure which is associated with delay \( \tau_k \), where \( \tau_k \) denotes the \( k \)th delay, \( k = 1, \ldots, K \) and \( 0 < \tau_1 < \cdots < \tau_K < + \infty \).

The initial condition is \( u(s) = \varphi(s) \), for \( s \in [-\tau, 0] \), where \( \varphi \in C([-\tau, 0], \Omega) \). Here, \( C([-\tau, 0], \Omega) \) denotes the Banach space of continuous vector-valued functions mapping the interval \([-\tau, 0]\) into \( \mathbb{R}^n \) with a topology of uniform convergence.

Considering the influences of uncertainties for model (1), we can describe (1) as

\[
\dot{u}(t) = -\varphi(u(t)) \left[ \beta(u(t)) - (T_0 + \Delta T_0)G(u(t)) - \sum_{k=1}^{K} (T_k + \Delta T_k)G(u(t - \tau_k)) + D \right],
\]

(2)

where \( \Delta T_k = [\Delta t_{kij}] \in \mathbb{R}^{m \times n}, k = 0, \ldots, K \) are time-invariant matrices representing the norm-bounded uncertainties. Now, the interconnecting matrix \( T = \sum_{k=1}^{K} (T_k + \Delta T_k) \) is nonsymmetric due to the perturbation terms \( \Delta T_k, k = 0, \ldots, K \).

Throughout this paper, we have the following assumptions.

**Assumption 1.** For the uncertainties \( \Delta T_k, k = 0, \ldots, K \), we assume

\[
[\Delta T_0 \ldots \Delta T_K] = HF[U_0 \ldots U_K],
\]

(3)

where \( F \) is an unknown matrix representing parametric uncertainty, which satisfies

\[
F^TF \preceq I,
\]

(4)

where \( I \) is an identical matrix, and \( H, U_0, \ldots, U_K \) can be regarded as the known structural matrices of uncertainty with appropriate dimensions.

**Assumption 2.** For \( i = 1, \ldots, n \), we assume

\( (H_1) \beta_i(u_i) \) is continuous and differentiable, and satisfies

\[
\beta_i'(u_i) \geq \beta_i''(u_i) > 0.
\]

(5)

\( (H_2) g_i(u_i) \) is bounded and satisfies

\[
0 \leq \frac{g_i(x) - g_i(y)}{x - y} \leq c_i, \quad \forall x, y \in \mathbb{R} \text{ with } x \neq y.
\]

(6)
**Definition.** The equilibrium point of system (1) is said to be globally robustly stable with respect to the perturbation $\Delta T_k$, $k = 0, \ldots, K$, if the equilibrium point of system (2) is globally asymptotically stable.

Without loss of general, we suppose $u_c = (u_{c1}, \ldots, u_{cn})^T$ is an equilibrium point of network (2), then by means of coordinate translation $x = u - u_c$, we can obtain the new description of the neural networks (2)

$$\dot{x}(t) = -A(x(t)) \left[ B(x(t)) - (T_0 + \Delta T_0)S(x(t)) - \sum_{k=1}^{K}(T_k + \Delta T_k)S(x(t - \tau_k)) \right],$$

(7)

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$,

$$A(x) = \text{diag}[a_1(x_1), \ldots, a_n(x_n)], \quad a_i(x_i) = a_i(x_i + u_{ci}),$$

(8)

$$B(x) = [b_1(x_1), \ldots, b_n(x_n)]^T, \quad b_i(x_i) = b_i(x_i + u_{ci}) - b_i(u_{ci}),$$

(9)

$$S(x) = [s_1(x_1), \ldots, s_n(x_n)]^T, \quad s_i(x_i) = g_i(x_i + u_{ci}) - g_i(u_{ci}).$$

(10)

From the above Assumption 2, we have

$$b_i(x_i)/x_i \geq \beta_i > 0,$$

(11)

$$0 \leq s_i(x_i)/x_i \leq \sigma_i$$

(12)

for $i = 1, \ldots, n$. From (9) and (10), the origin is an equilibrium point of (7). Thus, we shift the equilibrium $u_c$ to the origin. In order to study the global robust stability of the equilibrium point for (1), it suffices to investigate the globally asymptotic stability of equilibrium point $x = 0$ of system (7).

**Lemma (Singh [15]).** If $U$, $V$ and $W$ are real matrices of appropriate dimension with $M$ satisfying $M = M^T$, then

$$M + UVW + W^TV^TU^T < 0$$

(13)

for all $V^TV \leq I$, if and only if there exists a positive constant $\varepsilon$ such that

$$M + \varepsilon^2 UU^T + \varepsilon W^TVW < 0.$$  

(14)

In the following section, we will give the sufficient conditions for the globally asymptotically stable of equilibrium point $x = 0$ of system (7).

3. Robust stability

**Theorem.** For any bounded delay $\tau_k$, $k = 1, \ldots, K$, the equilibrium point $x = 0$ of system (7) is globally asymptotically stable, i.e., the equilibrium point of system (1) is globally robustly stable with respect to the perturbation $\Delta T_0$ and $\Delta T_k$, $k = 1, \ldots, K$, if there exist a positive definite diagonal matrix $Q = \text{diag}(q_1, \ldots, q_n)$, $q_i > 0$, $i = 1, \ldots, n$, a positive constant $\varepsilon$, and positive definite symmetric matrices $T_k$, $k = 1, \ldots, K$, such that the following LMI holds:

$$\begin{bmatrix}
\phi & QT_k + \varepsilon U_0^T U_K & \cdots & \cdots & QT_0 + \varepsilon U_0^T U_0 & QH \\
T_k^T Q + \varepsilon U_0^T U_0 & -\Gamma_k + \varepsilon U_0^T U_K & \cdots & \cdots & \varepsilon U_0^T U_0 & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
H^T Q & 0 & \cdots & \cdots & -\Gamma_0 + \varepsilon U_0^T U_0 & 0 \\
\end{bmatrix} < 0,$$

(15)

where $\phi = -2QB^mE^{-1} + QT_0 + T_0^T Q + \sum_{k=1}^{K}\Gamma_k + \varepsilon U_0^T U_0$, $B^m = \text{diag}([\beta_1^m, \ldots, \beta_n^m])$ and $E = \text{diag}([\sigma_1, \ldots, \sigma_n])$.

**Proof.** Here, we introduce the following Lyapunov functional:

$$V(x) = 2 \sum_{i=1}^{n} q_i \int_0^\infty s_i(\xi) \frac{a_i(\xi)}{a_i(\xi)} d\xi + \sum_{k=1}^{K} \int_{t-\tau_k}^{t} S^T(x(\xi)) \Gamma_k S(x(\xi)) d\xi.$$  

(16)
The derivative of $V(x)$ with respect to $t$ along any trajectory of system (7) is given by

$$
\dot{V}(x) = -2S(x)^TQB^m x + 2S(x)^TQ(T_0 + \Delta T_0)S(x) + 2S(x)^TQ\sum_{k=1}^{K}(T_k + \Delta T_k)S(x(t - \tau_k))
$$

$$
+ \sum_{k=1}^{K}[S^T(x(t))\Gamma_k S(x(t)) - S^T(x(t - \tau_k))\Gamma_k S(x(t - \tau_k))]
$$

$$
\leq -2S(x)^TQB^m E^{-1}S(x) + 2S(x)^TQ(T_0 + \Delta T_0)S(x)
$$

$$
+ \sum_{k=1}^{K}S(x)^TQ(T_k + \Delta T_k)\Gamma_k^{-1}(T_k + \Delta T_k)^T Q S(x) + \sum_{k=1}^{K}S^T(x(t))\Gamma_k S(x(t))
$$

$$
= S^T(x)[-2QB^m E^{-1} + Q(T_0 + \Delta T_0) + (T_0 + \Delta T_0)^T Q + \sum_{k=1}^{K}[Q(T_k + \Delta T_k)\Gamma_k^{-1}(T_k + \Delta T_k)^T Q + \Gamma_k]]S(x)
$$

$$
= S^T(x)NS(x),
$$

(17)

where

$$
N = -2QB^m E^{-1} + Q(T_0 + \Delta T_0) + (T_0 + \Delta T_0)^T Q + \sum_{k=1}^{K}[Q(T_k + \Delta T_k)\Gamma_k^{-1}(T_k + \Delta T_k)^T Q + \Gamma_k].
$$

Thus, $\dot{V}(x)<0$ if $N<0$. By the functional differential equations theory [13], for any bounded delay $\tau_k>0$, $k = 1, \ldots, K$, the equilibrium point $x = 0$ of system (7) is globally asymptotically stable.

According to the famous Schur Complement [1], $N<0$ can be expressed by the following LMI:

$$
\begin{bmatrix}
\phi & Q(T_K + \Delta T_K) & \cdots & \cdots & \cdots & Q(T_0 + \Delta T_0) \\
(T_K + \Delta T_K)^T Q & -\Gamma_K & 0 & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \cdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \cdots & \ddots & \ddots & 0 \\
(T_0 + \Delta T_0)^T Q & 0 & \cdots & \cdots & 0 & -\Gamma_0
\end{bmatrix} < 0,
$$

(18)

where $\phi = -2QB^m E^{-1} + Q(T_0 + \Delta T_0) + (T_0 + \Delta T_0)^T Q + \sum_{k=0}^{K}\Gamma_k$. In fact, (18) is exactly

$$
\begin{bmatrix}
-2QB^m E^{-1} + QT_0 + T_0^T Q + \sum_{k=0}^{K}\Gamma_k & QT_K & \cdots & \cdots & \cdots & QT_0 \\
T_K^T Q & -\Gamma_K & 0 & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \cdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \cdots & \ddots & \ddots & 0 \\
T_0^T Q & 0 & \cdots & \cdots & 0 & -\Gamma_0
\end{bmatrix}
$$
Because of $[\Delta T_0 \ldots \Delta T_K] = H[F_0 \ldots F_K]$, (19) can be expressed as

$$
\begin{bmatrix}
Q \Delta T_0 + \Delta T_0^T Q & Q \Delta T_K & \ldots & \ldots & Q \Delta T_0 \\
\Delta T_K^T Q & 0 & \ldots & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\Delta T_0^T Q & 0 & \ldots & \ldots & 0
\end{bmatrix} < 0.
$$

Using the lemma, we know that (20) holds for all $F^T F \leq I$ if and only if there exists a constant $\varepsilon > 0$ such that

$$
\begin{bmatrix}
Q H^T Q & 0 & \ldots & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & 0 \end{bmatrix} + \frac{1}{\varepsilon} \begin{bmatrix}
U_0^T U_0 & U_0^T U_K & \ldots & \ldots & U_0^T U_0 \\
U_K^T U_0 & U_K^T U_K & \ldots & \ldots & U_K^T U_0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & \ldots & 0
\end{bmatrix} < 0.
$$
Rearranging (21), we get
\[
\begin{bmatrix}
\gamma & QT_K + \varepsilon U_0^T U_K & \cdots & \cdots & QT_0 + \varepsilon U_0^T U_0 \\
T_k^T Q + \varepsilon U_k^T U_0 & -\Gamma_K + \varepsilon U_k^T U_K & \cdots & \cdots & \varepsilon U_0^T U_0 \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
T_0^T Q + \varepsilon U_0^T U_0 & \varepsilon U_0^T U_K & \cdots & \cdots & -\Gamma_0 + \varepsilon U_0^T U_0 \\
\end{bmatrix}
< 0,
\]
where \( \gamma = -2QB^mE^{-1} + QT_0 + T_0^T Q + \sum_{k=0}^{K} \Gamma_k + \frac{1}{2}QHH^T Q + \varepsilon U_0^T U_0 \). By use of Schur Complement, (22) is equivalent to condition (15). This proves the globally asymptotic stability of the equilibrium point \( x = 0 \) for system (7), i.e., the equilibrium point of system (1) is globally robustly stable with respect to the uncertainties \( \Delta T_k, k = 0, \ldots, K \). \( \square \)

When \( \Delta T_k = 0, k = 0, \ldots, K \), we have the following corollary.

**Corollary 1.** For any bounded delay \( \tau_k, k = 1, \ldots, K \), the equilibrium point \( x = 0 \) of system (7) is globally asymptotically stable, if there exist a positive definite diagonal matrix \( Q = \text{diag}(q_1, \ldots, q_n), q_i > 0, i = 1, \ldots, n \), and positive definite symmetric matrices \( \Gamma_k, k = 1, \ldots, K \), such that the following LMI holds:
\[
\begin{bmatrix}
-2QB^mE^{-1} + QT_0 + T_0^T Q + \sum_{k=0}^{K} \Gamma_k & QT_K & \cdots & \cdots & QT_0 \\
T_k^T Q & -\Gamma_K & 0 & \cdots & 0 \\
\vdots & 0 & \ddots & \cdots & \vdots \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
T_0^T Q & 0 & \cdots & 0 & -\Gamma_0 \\
\end{bmatrix}
< 0.
\]

The corollary follows in a straightforward manner by choosing \( \Delta T_k = 0, k = 0, \ldots, K \) in (18). By using of the Lemma \((K + 1)\) times, we can obtain the following corollary.

**Corollary 2.** For any bounded delay \( \tau_k \), the equilibrium point \( x = 0 \) of system (7) is globally asymptotically stable, if there exist a positive definite diagonal matrix \( Q = \text{diag}(q_1, \ldots, q_n), q_i > 0, i = 1, \ldots, n \), positive constants \( \varepsilon_k, k = 0, \ldots, K \), and positive definite symmetric matrices \( \Gamma_k, k = 1, \ldots, K \), such that the following LMI holds:
\[
\begin{bmatrix}
\phi & QT_K & \cdots & QT_0 + \varepsilon U_0^T U_0 & QH & \cdots & \cdots & QH \\
T_k^T Q & -\Gamma_K + \varepsilon_k U_k^T U_K & 0 & \cdots & 0 & \varepsilon_k I & \cdots & \cdots \\
\vdots & 0 & \ddots & \cdots & \vdots & \vdots & \ddots & \cdots \\
\vdots & \vdots & \ddots & \cdots & \vdots & \vdots & \ddots & \cdots \\
T_0^T Q + \varepsilon_0 U_0^T U_0 & 0 & \cdots & 0 & -\Gamma_0 + \varepsilon U_0^T U_0 & 0 & \cdots & \cdots & 0 \\
H^T Q & 0 & \cdots & \cdots & 0 & -\varepsilon K I & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \cdots & \vdots & \vdots & \ddots & \ddots & 0 \\
\vdots & \vdots & \ddots & \cdots & \vdots & \vdots & \ddots & \ddots & 0 \\
H^T Q & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & -\varepsilon_0 I \\
\end{bmatrix}
< 0,
\]
where \( \phi = -2QB^mE^{-1} + QT_0 + T_0^T Q + \sum_{k=0}^{K} \Gamma_k + \varepsilon U_0^T U_0 \).
Proof. By using (3), inequality (19) can be rewritten as

\[
\begin{bmatrix}
-2QB^mE^{-1} + QT_0 + T_0^TQ + \sum_{k=0}^{K} \Gamma_k QT_k & \cdots & \cdots & \cdots & QT_0 \\
T_k^TQ & -\Gamma_k & 0 & \cdots & 0 \\
\vdots & & 0 & \ddots & \vdots \\
\vdots & & \vdots & \ddots & \vdots \\
\vdots & & \vdots & \vdots & 0 \\
T_0^TQ & 0 & \cdots & \cdots & 0 -\Gamma_0
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
QH \\
0 \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
U_k^T \\
\vdots \\
0
\end{bmatrix}
+ \begin{bmatrix}
F[0 U_K 0 \cdots 0] \\
F[0 U_K 0 \cdots 0] \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
F^T[H^TQ 0 \cdots 0] \\
F^T[H^TQ 0 \cdots 0] \\
\vdots \\
0
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
QH \\
0 \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
U_0^T \\
0 \\
\vdots \\
0
\end{bmatrix}
+ \begin{bmatrix}
F[U_0 0 \cdots 0 U_0] \\
F[U_0 0 \cdots 0 U_0] \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
F^T[H^TQ 0 \cdots 0] \\
F^T[H^TQ 0 \cdots 0] \\
\vdots \\
0
\end{bmatrix} < 0.
\]

Using the lemma \((K + 1)\) times, (25) changes to

\[
\begin{bmatrix}
-2QB^mE^{-1} + QT_0 + T_0^TQ + \sum_{k=0}^{K} \Gamma_k QT_k & \cdots & \cdots & \cdots & QT_0 \\
T_k^TQ & -\Gamma_k & 0 & \cdots & 0 \\
\vdots & & 0 & \ddots & \vdots \\
\vdots & & \vdots & \ddots & \vdots \\
\vdots & & \vdots & \vdots & 0 \\
T_0^TQ & 0 & \cdots & \cdots & 0 -\Gamma_0
\end{bmatrix}
\]
Rearranging (26), we get
\[
\begin{bmatrix}
  QHH^T Q & 0 & \cdots & 0 \\
  0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & 0 \\
\end{bmatrix} + \frac{1}{e_K}
\begin{bmatrix}
  0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & 0 \\
\end{bmatrix} + e_K
\begin{bmatrix}
  0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & 0 \\
\end{bmatrix} < 0.
\]

For (28), we choose the parameters as
\[
T_0 = \begin{bmatrix}
  0.5330 \\
  1.0328 
\end{bmatrix}, \quad T_1 = \begin{bmatrix}
  -0.0184 \\
  -0.6138 
\end{bmatrix}.
\]

\[
S(x(t)) = \begin{bmatrix}
  \tanh(0.5x_1), \\
  \tanh(0.5x_2)
\end{bmatrix}^T.
\]

4. Illustrative example

We will use MATLAB’s LMI toolbox and Simulink modules for the simulation of system (7) to verify the effectiveness of our results. Considering the case that there exist a delay \(\tau_1\), system (7) can be rewritten as

\[
\dot{x}(t) = -A(x(t))[B(x(t)) - (T_0 + \Delta T_0)S(x(t)) - (T_1 + \Delta T_1)S(x(t - \tau_1))].
\]

For (28), we choose the parameters as
\[
T_0 = \begin{bmatrix}
  0.5330 & -1.0520 \\
  1.0328 & 0.3621 
\end{bmatrix}, \quad T_1 = \begin{bmatrix}
  -0.1844 & -0.1375 \\
  -0.6138 & -0.0802 
\end{bmatrix}.
\]

\[
S(x(t)) = [\tanh(0.5x_1), \tanh(0.5x_2)]^T.
\]

By linearizing \(S(x(t))\) at the origin, we have \(E = \text{diag}[0.5, 0.5]\). For the simplification, we choose \(B(x(t)) = [3, 4]^T, A(x(t)) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\), and \(F(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\). So, we have \(B^n = \text{diag}[3, 4]\).
From Assumption 1, we have $[\Delta T_0 \Delta T_1] = HF[U_0 U_1]$, where

$$U_0 = \begin{bmatrix} 0.2500 & 0.5000 \\ -1.0000 & -0.5000 \end{bmatrix}, \quad U_1 = \begin{bmatrix} -0.5000 & 1.5000 \\ 1.0000 & 0.1000 \end{bmatrix}, \quad H = \begin{bmatrix} 0.2000 & 1.0000 \\ -1.2000 & -1.0000 \end{bmatrix}.$$

By solving the associated LMI using LMI toolbox, we know that there exists a positive diagonal matrix

$$Q = \begin{bmatrix} 0.2892 & 0 \\ 0 & 0.1924 \end{bmatrix},$$

two positive definite symmetric matrices

$$\Gamma_0 = \begin{bmatrix} 0.9290 & 0.1137 \\ 0.1137 & 0.7288 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0.9222 & -0.1248 \\ -0.1248 & 0.9307 \end{bmatrix}$$

and a positive constant $\varepsilon = 0.1731$, such that the LMI (15) holds, hence the equilibrium point $x = 0$ of system (28) is globally asymptotically stable for arbitrarily bounded delay $\tau_1$.

During the process of simulation, we choose the different initial functions. The simulation curves are presented in Figs. 1 and 2. When $\tau_1 = 1.5\text{s}$, the simulation curves of system converging to asymptotically stable equilibrium point $x = 0$ are presented in Fig. 1. When $\tau_1 = 2.3\text{s}$, the simulation curves are presented in Fig. 2.
By the above simulation results, we can know clearly that for any bounded delay $\tau_1$, the state curves of system converge to asymptotically stable equilibrium point $x = 0$.

5. Conclusion

During the implementation process of neural networks by electronic circuits, time delays and uncertainties are inevitable. The present paper analyzes the robust stability of a class of Cohen–Grossberg neural network model with multiple time delays and uncertainties, and establishes sufficient conditions for the global robust stability of equilibrium point under arbitrarily bounded delays $\tau_k$, $k = 1, \ldots, K$. The results of this paper take the form of LMI and are independent of the amplification function. Thus, they are very practical for the analysis and design of Cohen–Grossberg neural networks with delays.

In applications, the bounds of time delays are typically not very large and are usually known. Therefore, the next research work is to investigate further whether we can obtain sufficient conditions which depend on time delay $\tau_k$, $k = 1, \ldots, K$.

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